ALTERED TOOTH-SUM GEARING FOR HIGH CONTACT RATIO

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ABSTRACT

In gearing smoothness in transmission of power depends mainly on the contact ratio. Contact ratio determines the load shared by the teeth in mesh and it will be lower when the contact ratio is higher. Therefore in high contact ratio gearing the load transfer from one tooth mesh to another takes place at a lower magnitude of load and the noise produced by this kind of a gearing will be lower. Profile modification in involute tooth gearing helps build properties to the gears in mesh that are uncommon to standard gears and further, it also helps in altering the tooth-sum of a gear pair operating between a specified centre distance and module. It is well known that highest contact ratio is obtained using standard gears is 1.94 (when a pinion meshes with a rack). Using altered tooth-sum gearing it is possible to achieve contact ratios higher than two. Thus high contact ratio gearing helps bring down the contact stresses and improve durability of gears. This paper presents the methods of using altered tooth-sum gearing to obtain high contact ratio and also discusses about its benefits.

.1. INTRODUCTION

Gear is the most widely used machine element whether it is for transmission of motion or for power. They are designed to meet various operating characteristics related to the type of tooth action or for strength requirements. More commonly as they are used for power transmission application in industry, design for strength becomes more important. Hence the power that the gear can transmit depends on the maximum permissible tooth load during the course of its mesh along the path of contact. The modern day industry requires gears that can transmit high power while maintaining high performance. In achieving this requirement it is necessary to reduce the tooth-load, which helps in lowering the tooth stresses, and hence life of the gears can be increased. Contact ratio is an important parameter, which is defined as the maximum number of teeth in mesh during the course of mesh along the path of contact. Therefore if more number of teeth are in mesh the tooth load correspondingly gets reduced which enables gears to transmit larger power without resorting to high strength materials or for a given material the gear size can be reduced.

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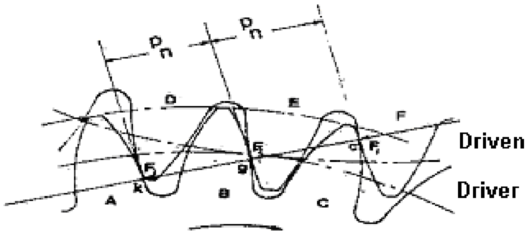


Fig.1 Tooth contact in high-contact ratio gears (point 'k' coincides with driven effective outside diameter)

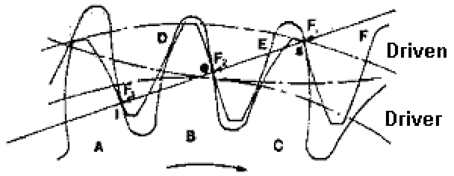


Fig.2 Tooth contact in high-contact ratio gears (point 'a' coincides with driver effective outside diameter)

A HCR gears in mesh are shown in Fig. 1, in which it is seen clearly that three teeth are in mesh and transmitting the load. The contact shown in Fig. 1 is such that the point ‘k’ on the effective outside diameter of the driven gear is the contact point between tooth ‘D’ on the driven gear and tooth ‘A’ on the driver. Points ‘c’ and ‘g’ are the other two points of contact as shown. The circular base pitch is denoted by the distance measured between the points ‘k-g’ or ‘g-c’ along the line of action. Fig. 2, represents the instant that occurs just after the one shown in Fig.1. In this case the point ‘a’ is the contact point that coincides with the effective outside diameter of the driver. Points ‘e’ and ‘i’ represent the other two points of contact. If the driving gear rotates in the direction indicated by the arrow as in Fig.2, the tooth ‘C’ will come out of mesh with the tooth ‘F’ and thus only two pairs of teeth will transmit the load. If the driver proceeds further, another tooth to the left of tooth ‘A’ will come into mesh with a corresponding tooth on the driven and three pairs of contact is repeated and the chain of events as described above will repeat.

2. EFFECTS ON CONTACT RATIO

Contact ratio of a gear pair is the ratio of the length of path of contact to the base pitch. The contact ratio is affected only when profile of the tooth is modified. In gearing, a contact ratio of 1.2 is taken as practical minimum in order to maintain smooth transfer of motion. Larger the contact ratio smoother will be the power transmission and also the noise level will be lower. Therefore it will be beneficial to have higher contact ratio. Standard gears (Uncorrected gears) in use have satisfactory contact ratio either for transfer of motion or power. In the most general case of gearing the contact ratio is found to be in the range of 1.6 to 1.7. Using standard gearing the highest contact ratio obtainable is 1.936 when a pinion meshes with a rack. But when gears of higher contact ratio are desirable the better option is to use profile corrected gears. Employing profile shifted gears of standard tooth-sum with their profile shift sum being equal to zero can achieve a maximum contact ratio of 1.8. However, the problem of achieving higher contact ratios of values greater than 2 can be met by using altered tooth-sum gearing. In this gearing a standard tooth-sum operating between a specified center distance and module can be altered by a value ±4 % of the tooth-sum as given in reference (4). Thus a tooth-sum of 100 and of module 2mm operating between a specified center distance of 100 mm can be altered by ±4 teeth and such alteration in tooth numbers will help accommodate tooth-sums ranging from 96 teeth to 104 teeth for the above center distance. Such alterations in tooth-sums operate on pressure angles different from the standard pressure angle. Hence for positive alterations in tooth numbers the operating pressure angle will be smaller than the standard pressure angle and vice versa. Also such changes as above necessitates profile shifting of gears in mesh. The following Table-1 gives the data of altered tooth-sum gearing. From this table it is evident that for positive alteration in tooth numbers the contact ratio obtained are greater than that available using standard tooth-sums.

Table-1: Details of altered tooth-sum gearing for a tooth-sum of 100 teeth and module 2mm.

No. of teeth Altered	Altered tooth-sum, Z'	No of teeth on pinion, Z_1'	No of teeth on the Gear Z_2'	Operating pressure angle Degree, α_s	Profile shift X_e	Profile shift on gears $X_1=X_2$	Contact ratio ε
0	100	50	50	20.00	0.00	0.00	1.75
1	101	50	51	18.36	-0.48	-0.24	1.84
2	102	51	51	16.56	-0.92	-0.46	1.93

3	103	51	52	14.56	-1.31	-0.65	2.01
4	104	52	52	12.24	-1.65	-0.82	2.08
-1	99	49	50	21.52	0.51	0.25	1.64
- 2	98	49	49	22.94	1.07	0.53	1.52
- 3	97	48	49	24.28	1.65	0.82	1.40
- 4	96	48	48	25.56	2.27	1.13	1.26

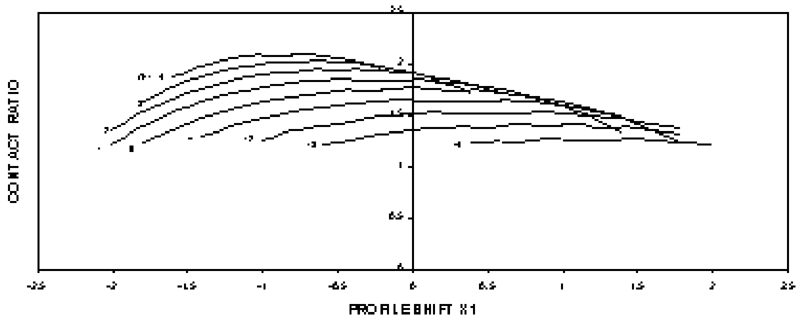


Fig. 3 VARIATION OF CONTACT RATIO WITH PROFILE SHIFT FOR Z=100 TEETH

To study the effects of varying the number of teeth on the contact ratio the tooth-sum of 100 may be considered. A gear ratio of 1:1 is used in all the cases and the tooth sum is varied from +4 teeth to –4 teeth and the profile shift for each value of number of teeth altered (Zi) is taken from the Table-1. The corresponding profile shift is allowed on the pinion in varying proportions such that the algebraic sum of the profile shift remains equal to the value of profile shift given in Table-1. Using a standard module of 2mm the geometrical dimensions are determined and contact ratio is calculated. Fig. 3 shows the variation of contact ratio for different values of Zi altered over the tooth-sum of 100 allowing the profile shift $x_1$on the pinion. It is seen in this figure that for positive values of teeth altered on the tooth-sum the contact ratio is higher and vice-versa. The values of contact ratio peak when the resulting profile shift due to alteration in tooth numbers is shared equally for gear ratio 1:1. This property can be used beneficially while designing gears, which is uncommon to standard gearing.

In Table-2 given below few cases of high contact ratio gearing for different tooth-sums ranging from 100 teeth to 180 teeth with an alteration in tooth numbers of +4 teeth indicate clearly that the contact ratio in all the cases given is greater than or equal to 2. Hence altered tooth-sum gearing can be employed to achieve higher contact ratios if it is so desirable.

Table 2: Gives the values of Highest Contact ratios in Altered tooth-sum Gearing.

Standard Tooth-sum	Z_1=100	Z_2=120	Z_3=140	Z_4=160	Z_5=180
Tooth-sum Altered by +4 teeth	Z_1^1 =104	Z_2^1=124	Z_3^1=144	Z_4^1=164	Z_5^1=184
Corresponding profile shift, X	-1.656	-1.721	-1.765	-1.797	-1.821
Profile shift on Pinion X1	Contact Ratio ε				
-1.2	2.02	2.01			
-1.1	2.05	2.02	2.01	2.00	
-1.0	2.06	2.03	2.01	2.00	2.00
-0.9	2.07	2.04	2.02	2.01	2.00
-0.8	2.08	2.04	2.02	2.00	2.00

-0.7	2.07	2.03	2.01	2.00	
-0.6	2.06	2.02	2.00		
-0.5	2.04	2.00			
-0.4	2.01				

3. Tooth load sharing

Gears when they are transmitting power the gear teeth in mesh transmit loads normal to their surfaces of contact. Along the path of contact the number of teeth in mesh transmitting this load depends on the contact ratio of the gear pair in mesh. Hence if the contact ratio is one a single pair of teeth transmit the entire load from the beginning of contact till the end of contact and if the contact ratio is between 1 and 2 for some duration of time along the path of contact two pairs of teeth transmit this load and for the rest single pair of teeth transmit this load. Thus, the load shared between the teeth in mesh along the path of contact varies from half load to full load. In this manner if the contact ratio is 2 the tooth load shared by the teeth in mesh along the entire length of path of contact will remain half the full load. Fig. 4 illustrates the tooth action along the path of contact beginning at point ‘A’ and ending at point ‘E’. The tooth load shared for different contact ratios is shown in Fig. 5(a) and 5(b).

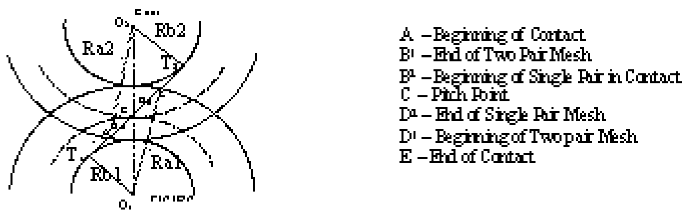
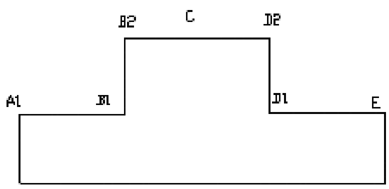


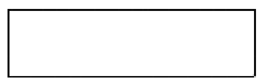
Fig.4 Path Of Contact

Fig. 4 Shows the path of contact for a pair of teeth in mesh from the beginning to the end described by the points A, B, C, D and E.



CONTACT RATIO >1.2 and < 2

Fig. 5(a)



CONTACT RATIO = 2

Fig. 5(b)

Fig. 5. a) Illustrates the load distribution between a pair of mating teeth along the path of contact. From A1 to B1 the load shared is half full load, From B2 to D2 full load and from D1 to E the load shared is half full load when the contact ratio is between 1.2 and 2.

b) Shows the load shared by a pair of mating teeth along the path of contact when the contact ratio is equal to 2.

4. EFFECT ON STRESSES

The loads acting on the gear tooth while transmitting power induce stresses in the material of the gear tooth, which are chiefly bending stresses and contact stresses. Between these two types of stresses contact stress prevailing in the region of tooth contact determines the durability of the gears. Nevertheless, bending stress is equally important but failure due to bending in power transmission gearing is not common. Hence consideration to the aspect of contact stresses becomes more prominent and in this discussion the effects of high contact ratio gearing on the contact stresses is presented. The effects of altering the tooth-sum on the contact stresses has been discussed in reference [5], in this paper the contact stress induced with respect to the amount of profile shift allowed on the pinion as well as on the gear has been presented (the profile shift allowed is due to the alteration in tooth numbers for a gear pair operating between a specified center distance and module. The amount of profile shift allowed on the gears is based on two important consideration, the first is to ensure a minimum contact ratio of 1.2 and the second is to ensure tooth tip thickness greater than 0.4 module). It is found that contact stresses are lower in the case of negatively altered tooth-sum gearing compared to that of positively altered tooth-sum gearing. Contact stress varies inversely as the radius of curvature of the tooth profile and in the negatively altered tooth-sum gearing the profile shift is positive and this results in lower contact stresses. On the other hand in the negatively altered tooth-sum gearing the contact ratio will be smaller than that in the case of positively altered tooth-sum gearing, which affects the tooth load sharing giving rise to larger contact stresses in the region of single pair tooth contact (which is predominant in this gearing). Where as in the case of positively altered tooth-sum gearing contact ratio being higher, results in better tooth load sharing and the contact stresses will be relatively lower although they receive negative profile shift. This effect is more pronounced in the case of high contact ratio gearing in which at any given point along the path of contact the tooth load shared is always equal to half the full load. Therefore high contact ratio gearing will have better surface durability compared to the standard gearing. The following Figs. 6(a) to 6(g) illustrate the contact stress variation with respect to the profile shift allowed on the pinion for the cases of altered tooth-sum gearing considered in Table-2, which give contact ratio greater than 2. (Number of teeth altered is +4 on the tooth-sum). These figures refer to the contact stress induced at various points such as 'A' to 'E' along the path of contact as shown in Fig. 4. In computing the contact stress in these cases a module of 2 mm is considered to determine the geometric dimensions and a tooth load of 10 Newton per millimeter width of the gear face is applied. In comparing the level of contact stresses induced in altered tooth-sum gearing vis-à-vis standard tooth-sum gearing or S_0 Gearing (In this gearing the tooth-sum is not altered but profile shift is allowed on the gears such that the sum of the profile shifts will be equal to zero or $x_1+x_2=0$). The contact stresses computed at various points as explained earlier have been shown for this gearing in Fig.7 (a), which represents the contact stress at the beginning of contact, at this point the tooth load shared is half the full load, and 7(b) represents the contact stress when the tooth load shared is full. These points have been selected because the contact stress is generally higher at these points as they are very close to the base circle and point B2 indicates the stress when the tooth load shared is full. In Table-3 the values of contact stresses are given for comparison. The high and low values of stresses refer to the highest and lowest values of stresses plotted in Fig. 6(a) and 6(c) and 7(a) and 7(b). From this table it is evident that the contact stresses in altered tooth-sum gearing are relatively lower than those for the standard tooth-sum gearing. This clearly demonstrates that altered tooth-sum gearing can be preferred over the standard tooth-sum gearing for lower contact stresses or for better durability.

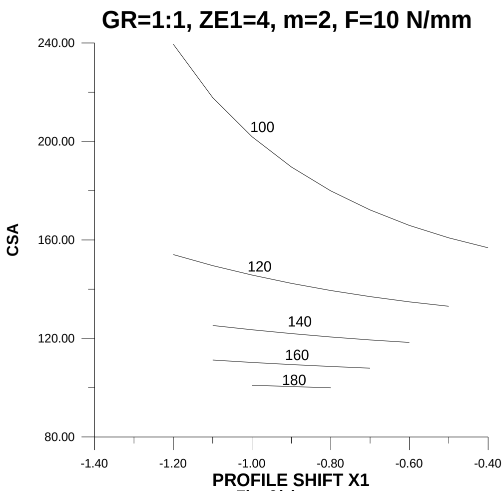


Fig. 6(a)

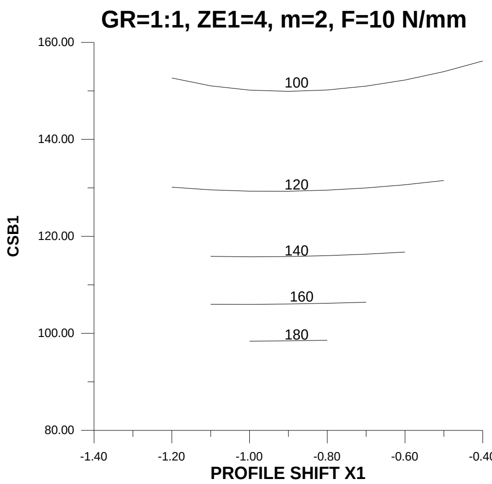


Fig. 6(b)

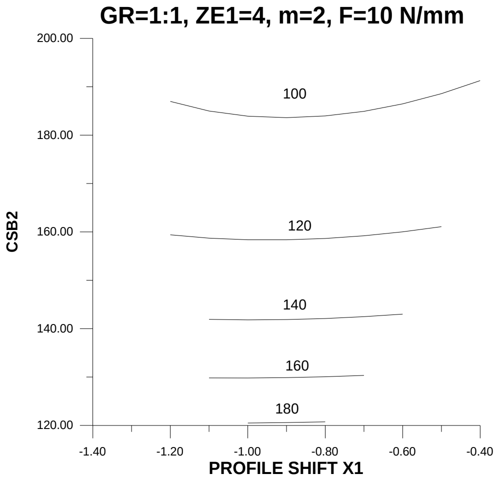


Fig. 6(c)

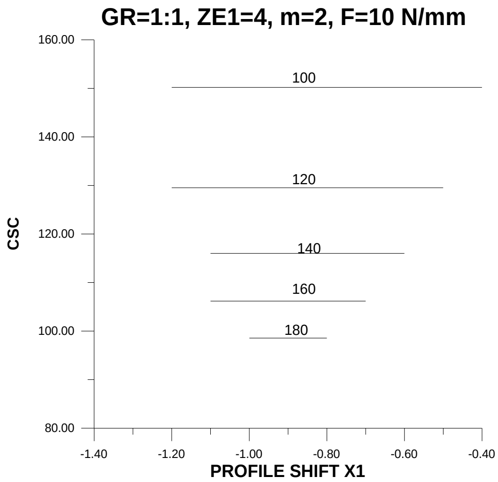


Fig. 6(d)

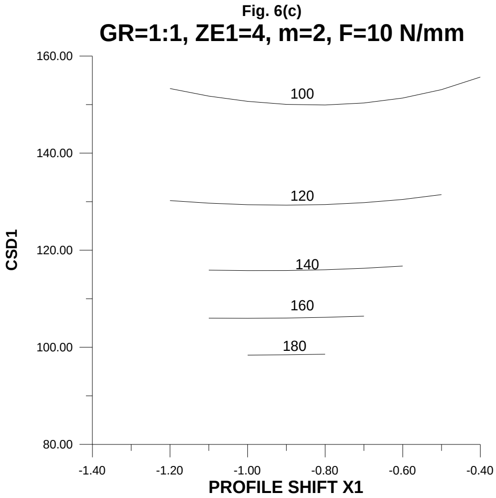


Fig. 6(e)

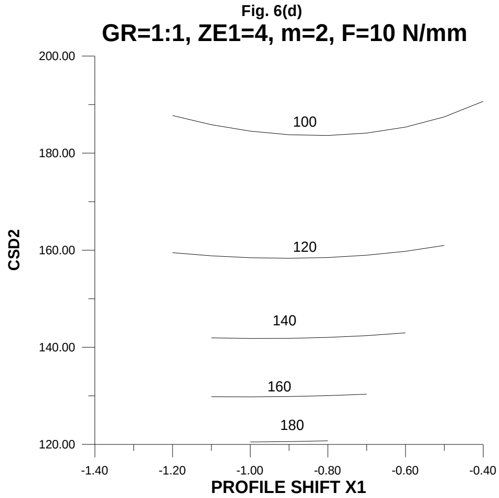


Fig. 6(f)

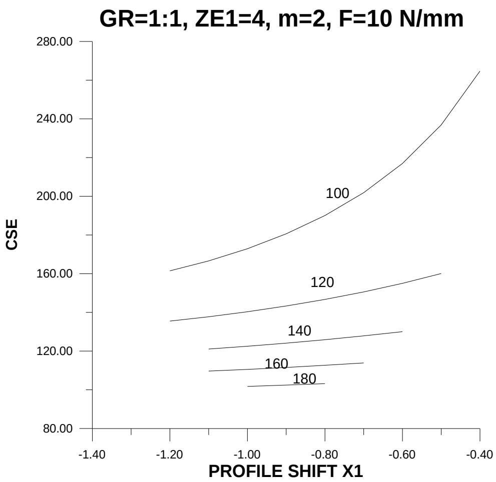


Fig. 6(g)

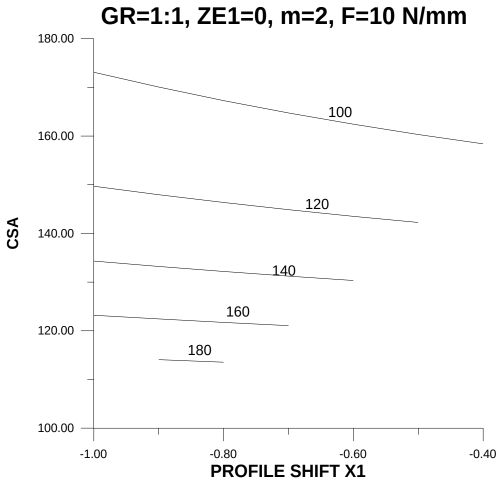


Fig. 7(a)

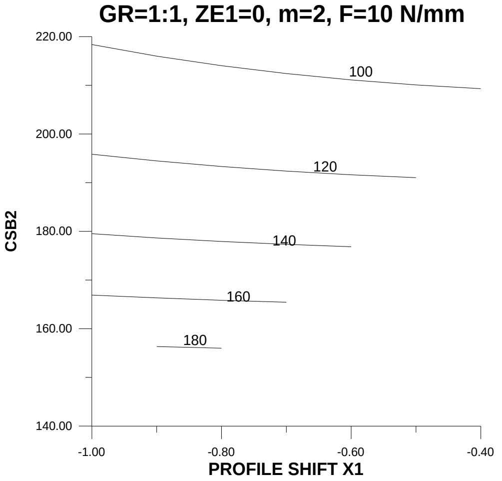


Fig. 7(b)

Table 3 Gives the details of contact stresses at points 'A' and 'B2' along the path of contact for altered tooth-sum gearing and the values shown in parenthesis are for the standard tooth-sum gearing.

Number of teeth in Altered Tooth-sum		104 (100)	124 (120)	144 (140)	164 (160)	184 (180)
Point 'A'	High, MPa	239.49 (173.12)	154.03 (149.69)	125.24 (134.31)	111.22 (123.18)	100.98 (114.06)
	Low, MPa	156.81 (158.39)	133.06 (142.24)	118.36 (130.33)	107.92 (121.05)	99.94 (113.54)
Point 'B2'	High, MPa	186.98 (218.38)	159.39 (195.84)	141.91 (179.51)	129.81 (166.90)	120.49 (156.33)
	Low, MPa	191.27 (209.31)	161.07 (191.02)	142.99 (176.84)	130.33 (165.43)	120.73 (155.99)

4. CONCLUSION

Contact ratio in gearing is an important parameter in gearing, which determines the maximum number of teeth in mesh along the path of contact. Higher the number of teeth in mesh lower is the tooth load. Obviously lower tooth load induces lower stresses or it is possible to load the tooth to a higher value for a given permissible stress. Thus high contact ratio becomes more relevant in power transmission gearing. The use of altered tooth sum gearing permits design gears for higher contact ratio. Following are the conclusion drawn based on this paper.

1) Based on the study contact ratios of equal to or greater than 2 are obtained for positive alterations in tooth number of value 4 teeth on the tooth-sums ranging from 100 teeth to 180 teeth. However, for the study tooth-sums ranging from 40 teeth to 240 teeth have been considered.
2) As contact ratio is higher the maximum tooth load shared by the tooth will not exceed more than half the full load along the path of contact.
3) Contact stresses computed for altered tooth-sum gearing are much lower than that for the standard tooth-sum gearing. Hence lower contact stresses permit larger power transmission capabilities.
4) As contact ratio in altered tooth-sum gearing is higher the noise produced while transmitting power will be lower.

5. REFERENCES

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